Respective Readings and Gaps

Jean Mark Gawron, SRI International[1]
Andrew Kehler, SRI International[1]
Linguistics Society of America

January 7, 2000

1 Introduction

An Analysis relating "Full" and "Reduced" Syntactic Structures:
(Pullum and Gazdar, 1982; Kac, 1987; Kay, 1989; Moltmann, 1992; Dalrymple and Kehler, 1995; McCawley, 1998; inter alia)

(1) a. *Tolstoy and Dostoyevsky* wrote *Anna Karenina and The Idiot* respectively.
b. *Tolstoy* wrote *Anna Karenina* and *Dostoyevsky* wrote *The Idiot*.

2 Syntactic Issues

Examples in which syntactically full forms are unavailable (Dalrymple and Kehler, 1995)

(2) Though the Trail Blazers won this series in six games from Phoenix, they were far from dominant. Their margins of victory were 2 points, 1 point, 6 points, and 3 points (respectively).

Respectively Reduction and Conjunction Reduction

(3) a. John and Mary met.
b. ? John met and Mary met.

Interaction with Gaps

(4) I finally met Susan, Marilyn, and Lucille yesterday. They are the three sisters that Bob married, John is engaged to, and Bill is dating, respectively.

(5) I wonder which three sisters Bob married, John is engaged to, and Bill is dating respectively?

Problems for the Syntax of Long-Distance Dependencies

1. Examples (4) and (5) are problematic for a movement account of long-distance dependencies.
2. They also raise issues for an account that coindexes fillers with gap sites.

Problems regarding the distribution of *respectively*

(6) a. * John married Alice and Bill dated Betty, respectively.[2]
b. * *Tolstoy and Dostoyevsky* wrote *Anna Karenina or The Idiot* respectively.

[1]Artificial Intelligence Center, 333 Ravenswood Avenue, Menlo Park, CA 94025, {gawron | kehler}@ai.sri.com
[2]Our thanks to Ivan Sag for drawing our attention to this issue.

3 Semantic Issues

Group-Denoting NPs in the *respectively* Construction

(7) a. Group NPs appear in the construction: (2), (4) and (5).
b. NPs like *Tolstoy and Dostoyevsky* in (1) are interpreted as ordinary group denoting NPs and and not as syntactically reduced forms.
c. For conjoined NPs like *Tolstoy and Dostoyevsky*, the surface form of the NP provides an ordering on the group.

(8) a. Joan read *Madame Bovary* and *Sentimental Education*. He detested the **latter**.
b. Joan read *War and Peace* and *The Idiot*, but not **in that order**.

Respectively Readings independent of "respectively"

(9) a. Mary has a classic Beetle, and Susan has an old BMW. They fix them up every weekend.
b. Rock music fans came to the concert by every conceivable means. They walked, drove, sailed, and flew.
c. Chance is the first baseman. Tinkers and Evers are the shortstop and the second baseman.
d. Bob and Barbara are husband and wife. (Link 1984)
e. Propositions and properties may be summed to form proposition groups and property groups.

4 Sketch of Analysis

(10) Sue and Karen jog and drive to work respectively.

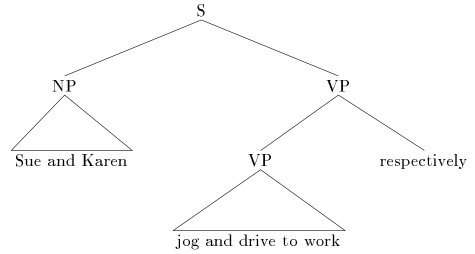

Figure 1: Tree for (10)

Sequencing the atoms in a group

(11) a. For capturing respectively readings, we use pragmatically supplied **sequencing functions**. A sequencing function f is a partial function from $\mathcal{N}$ to selector functions. A selector function is a function from groups to **atoms** of the group. For a sequencing function f and an individual sum g we write

$$f_i(g)$$

f^2	Group	
	k $\vee$ s	jog $\sqcap$ drive-to-work
f_1	s	jog
f_2	k	drive-to-work
f_3	$\uparrow$	$\uparrow$

Table 1: The sequencing function for example (10)

for the ith member of g under the sequencing $f(g)$.

b. We require, for any sequencing function f and sum g of cardinality n, that $f(g)$ give a one-to-one correspondence from indices less than or equal to n to g. If g is a sum of cardinality n and i is an index greater than n, $f_i(g)$ is undefined.

Respectively Readings

(12)

$$\bigwedge_{i=1}^{n} [f_i(\mathrm{P})](f_i(g))$$

We require that $n \geq 2$.

Example (10) using the sequencing function in Table 1

(13) $$\bigwedge_{i=1}^{2} [f_i(\text{jog} \sqcap \text{drive-to-work})](f_i(\text{k} \vee \text{s})) = \text{jog(k)} \sqcap \text{drive-to-work(s)}$$

How useful are property sums?

(14) a. Alice and Betty drove to Berkeley and Santa Cruz at the same time.
b. Alice and Betty met.

Claims of the Analysis

(15) a. Syntactically, *respectively* is an adverb.
b. Semantically, *respectively'* is polymorphic, of type $\langle$ X, X $\rangle$, the type of an adjunct:

$$\text{Resp} = \text{respectively'} = \lambda \mathrm{P} \lambda \mathrm{g} \bigwedge_{i=1}^{n} [f_i(\mathrm{P})](f_i(g))$$

Below, we show examples of *respectively* modifying VPs, TVPs, and other adverbials.

c. Respectively readings without respectively (9) are accounted for by free occurrence of the Resp operator.
d. A close analogy between the *respectively* and the distributivity operator (Link 1983, Roberts 1987):

$$\text{Dist} = \lambda \mathrm{P} \lambda g \bigwedge_{x \ \text{atom} \sqsubseteq_i g} \mathrm{P}(x)$$

Type	Sum	Atoms
individual	k $\vee$ s	{k, s}
proposition	walk(k) $\sqcap$ drive-to-work(s)	{walk(k), drive-to-work(s)}
property	walk $\sqcap$ drive-to-work	{walk, drive-to-work}

Table 2: Analogy between boolean sums and individual sums

1. Each is adverbial in type.
2. Each is a freely occurring operator which applies to a relation to give the next argument in a reading on which groups are distributed.
3. In each case, there is an adverb which can be translated as the operator (*each*, *respectively*).

4.1 Property Sums and Generalized Conjunction as Generalized Join

1. Relation conjunction and proposition conjunction treated, on analogy with individual plurals, as a JOIN (or summing) operation. (but still written $\sqcap$).
2. Propositions are primitives. Relations are functions from entities to propositions.
3. Just as individuals may be summed to form group objects, propositions and properties may be summed to form group propositions and group properties. (See appendix.) Sum-formation for both individuals and properties is linked to conjunction, which is why *or* is blocked in the *respectively* construction.
4. Montagovian denotations are of no help here:

$$[\![\lambda x[\text{walk}(x) \wedge \text{run}(x)]]\!] = \{\text{M, B}\}$$

4.2 Ways of Obtaining Property Sums

Property Conjunction (see (13))

Relation Conjunction and Right Node Raising

(16) a. George and Martha respectively denounced and were denounced by the governor. (McCawley 1998)
b. Resp $_{f^2}$ ([denounce$\sqcap$be-denounced-by](gov)) (George$\vee$Martha)

Proposition conjunction and lambda-abstraction

(17) a. $[\![$John was married to s and Bill was dating s$]\!]$ = [married-to(J, s)] $\sqcap$ [date(B, s)]
b. λs[married-to(J, s) $\sqcap$ date(B, s)]

Important Equation (true by (35d) in Appendix)

$$\lambda s\,[\text{married-to}(\text{J}, s) \sqcap \text{date}(\text{B}, s)] = \lambda s.\text{married-to}(\text{J}, s) \sqcap \lambda s.\text{date}(\text{B}, s)$$

Distributivity Operator

(18) a. Dist (see) = $\lambda g \bigwedge_{x \ \text{atom} \sqsubseteq_i g} \text{see}(x)$

b. $\mathrm{Dist\ (see)\ (J \vee M)} = \bigwedge_{x \ \mathrm{atom} \sqsubseteq_i \mathrm{J \vee M}} \mathrm{see}(x) = \mathrm{see(J)} \sqcap \mathrm{see(M)}$

Forming Property Sums (Summary)
Property sums can be formed whenever the widest-scoping operator in a VP denotation is conjunction, or (by lambda-abstraction) whenever the widest-scoping operator in an S denotation is conjunction.

4.3 Examples

(19) Tolstoy and Dostoyevsky wrote *Anna Karenina* and *The Idiot* respectively.

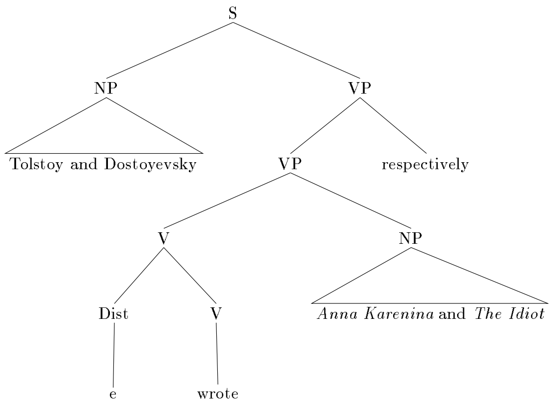

Figure 2: Analysis of (19)

V-Projections from Figure 2

(20) a. $[\![$ [Dist wrote] Anna K and The Idiot $]\!]$
$= \mathrm{Dist\ write(AK \vee I)}$
$= \mathrm{write(AK)} \sqcap \mathrm{write(I)}$

b. $[\![$ [Dist wrote] Anna K and The Idiot respectively $]\!]$
$= \mathrm{Resp}_{f^2}(\mathrm{write(AK)} \sqcap \mathrm{write(I)})$

c. $[\![$ Tolstoy and Dostoyevsky [Dist wrote] Anna K and The Idiot respectively $]\!]$
$= \lambda \mathrm{P.P(T \vee D)}(\lambda x.\mathrm{Resp}_{f^2}(\lambda w.\mathrm{write(AK}, w) \sqcap \mathrm{write(I}, w))(x))$
$= \mathrm{Resp}_{f^2}(\lambda w.\mathrm{write(AK}, w) \sqcap \mathrm{write(I}, w))(\mathrm{T \vee D})$
$= \mathrm{write(AK)(T)} \sqcap \mathrm{write(I)(D)}$

Double Respectively Readings

(21) a. George and Martha sent a bomb and a nasty letter respectively to the president and the governor respectively. (McCawley 1998)

b. George and Martha sent a bomb and a nasty letter to the president and the governor respectively.

c. Tolstoy and Dostoyevsky sent *Anna Karenina* and *The Idiot* to Dickens and Thackeray respectively.

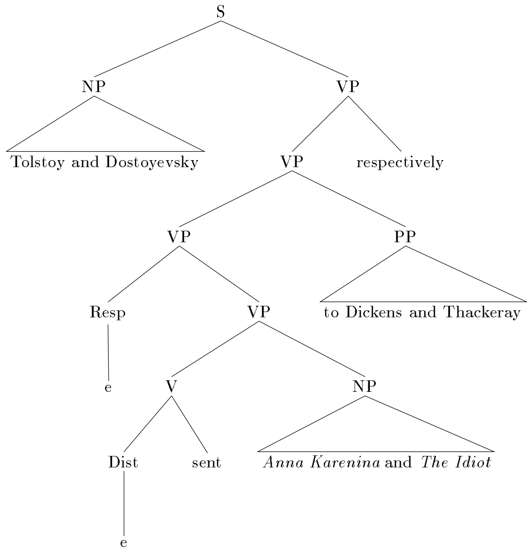

Figure 3: Analysis of (21c)

Major nodes from Figure 3

(22) a. $[\![$ [Dist sent] Anna K and The Idiot $]\!]$
$= \mathrm{Dist\ send(AK \vee I)}$
$= \mathrm{send(AK)} \sqcap \mathrm{send(I)}$

b. $[\![$ [Dist sent] Anna K and The Idiot respectively $]\!]$
$= \mathrm{Resp}_{f^2}(\mathrm{send(AK)} \sqcap \mathrm{send(I)})$

c. $[\![$ [Dist sent] Anna K and The Idiot to Dickens and Thackeray respectively $]\!]$
$= \lambda \mathrm{P.P(Di \vee Th)}(\lambda x.\mathrm{Resp}_{f^2}(\lambda w.\mathrm{send(AK}, w) \sqcap \mathrm{send(I}, w))(x))$
$= \mathrm{Resp}_{f^2}(\lambda w.\mathrm{send(AK}, w) \sqcap \mathrm{send(I}, w))(\mathrm{Di \vee Th})$
$= \mathrm{send(AK)(Di)} \sqcap \mathrm{send(I)(Th)}$

d. $[\![$ Tolstoy and Dostoyevsky [Dist sent] Anna K and The Idiot to Dickens and Thackeray respectively $]\!]$
$= \lambda \mathrm{P.P(T \vee D)}(\mathrm{Resp}_{f^2})(\mathrm{send(AK)(Di)} \sqcap \mathrm{send(I)(Th)})$
$= (\mathrm{Resp}_{f^2}(\mathrm{send(AK)(Di)} \sqcap \mathrm{send(I)(Th)})(\mathrm{T \vee D})$
$= \mathrm{send(AK)(Di)(T)} \sqcap \mathrm{send(I)(Th)(D)}$

(23) John and Mary drive to Santa Cruz and Berkeley on Tuesdays and Thursdays respectively.

Major nodes from Figure 4

(24) a. [[Dist on] Monday and Tuesday] $\longrightarrow$ [on(M)] $\sqcap$ [on(T)]

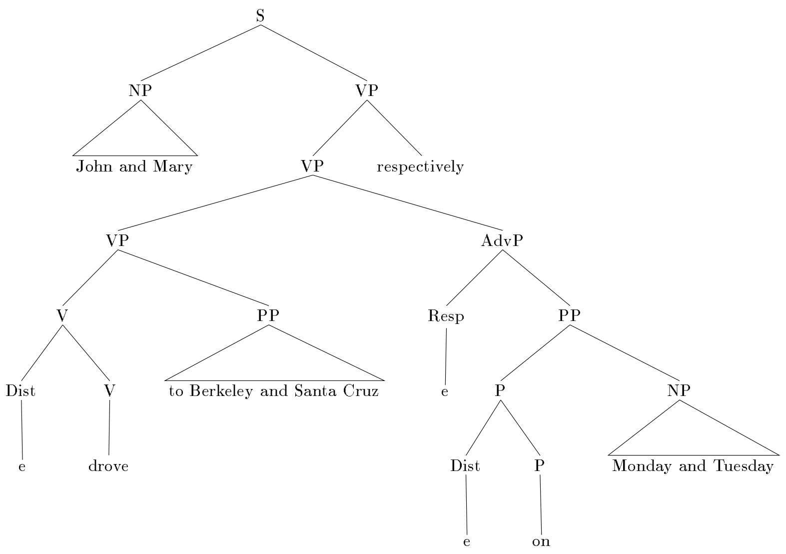

Figure 4: Analysis of (23)

b. [[Dist drove] to Berkeley and Santa Cruz] $\longrightarrow$ drive-to(B) $\sqcap$ drive-to(SC)

c. [[Dist drove] to Berkeley and Santa Cruz] [[Dist on] Monday and Tuesday]
$\longrightarrow$ Resp $_f$ ([on(M)] $\sqcap$ [on(T)]) (drive-to(B) $\sqcap$ drive-to(SC))
= [on(M)](drive-to(B)) $\sqcap$ [on(T)](drive-to(SC))

d. [John and Mary [Dist drove] to Berkeley and Santa Cruz] [[Dist on] Monday and Tuesday]
$\longrightarrow$ Resp $_f$ ([on(M)](drive-to(B)) $\sqcap$ [on(T)](drive-to(SC)) (J$\sqcap$M)
= [on(M)](drive-to(B))(J) $\sqcap$ [on(T)](drive-to(SC))(M)

5 Gaps

(25) Fred wondered which sisters John was married to and Bill was dating, respectively.

(26) wonder(f, which s sisters(s),
[Resp $_f$(λx [married-to(J, x) $\sqcap$ date(B, x)]) (s)])

What does respectively attach to in Gap examples?

(27) * John married Alice and Bill dated Betty, respectively.

Account

1. Semantic type of *respectively* rules out (27).
2. Gap containing constituents denote properties, which are of the right type for *respectively* (for example, Figure 5).

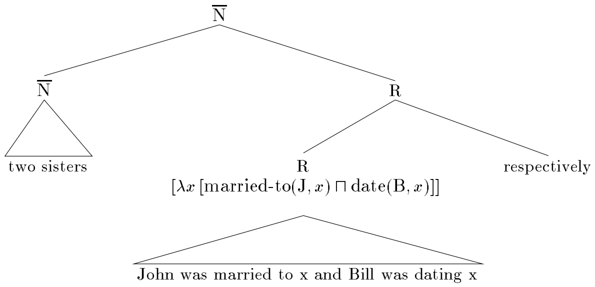

Figure 5: Relative clauses as property-denoting

Other adverbials and gaps

(28) a. John wrote a song and Mary sang it for free.
b. That's the song John wrote and Mary sang for free. [distributed reading]
c. What song did John write and Mary sing for free?

Scope Facts

1. Both the *date* relation and the *married-to* relation must be in the scope of *respectively* in (25).
2. *Respectively* must take scope lower than *which two sisters*.

The Problem for co-indexing

1. The variable of the filler cannot be unified with the binding variable as it is, for example, in the HPSG analysis (See Figure 6).
2. Resp is irreduceably a higher-order operator (it sums atoms in the denotation of its property argument).

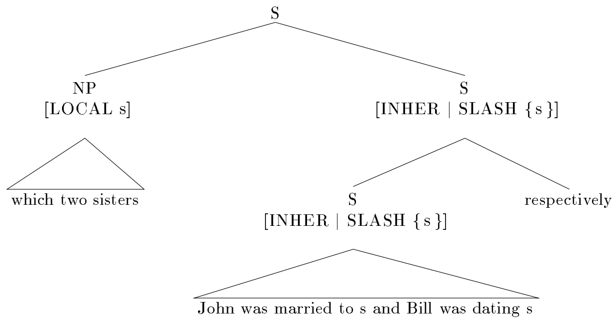

Figure 6: HPSG-style Treatment of respectively and Filler-Gap constructions

6 Conclusion

Syntax/Semantics Interface Consequences

(29) a. Feature passing principles like the HPSG Binding Inheritance Principle (Pollard and Sag 1994) remain unaffected.
b. Gap-containing constituents must at some point in the tree be interpreted as properties.
c. The relation between filler and gap involves a predication relation between the gap-containing S (a property) and the filler (its argument).
d. In unification-based approaches, there is an issue about using unification to capture predication relations. In *Kim and Sandy jog and drive to work respectively*, the analysis fails if the index for *Kim and Sandy* is unified with either the lexical subject role for *jog* or the lexical subject role for *drive*.

Summary

1. The conjunction *and* is always treated as a summing operation. (*Or* is not).
2. Proposition and relation sums are developed and motivated on analogy with individual sums.
3. The analysis provides motivation for primitive propositions. Interestingly, the relevant construction isn't intensional.
4. Much of the development here uses apparatus motivated independently for plurals (distributivity operator) and adverbial interpretation (property sums).

7 Appendix: Background for Semantics

A complete atomic semi-lattice on individuals (Link 1983)

(30) a. A transitive, reflexive, antisymmetric ordering $\sqsubseteq_i$ on the set of entities E.
b. For any subset S of E, we require the "sum" of the elements of S to be another element of E, written

$$\bigwedge_{e \in s} e$$

We define this individual to be the supremum (or join or least upper bound or l.u.b.) of S with respect to $\sqsubseteq_i$. We also call such an individual an *individual sum*. Since E is a set such that the sum of any subset is in E, E is a *complete join semilattice* with respect to $\sqsubseteq_i$. We write

$$\mathrm{a} \vee \mathrm{b}$$

for the supremum of a and b.
c. We define A_E, a subset of E, to be the set of **atoms** and to consist of those elements a such that

$$\forall p[p \sqsubseteq_i a \longrightarrow p = a]$$

We write

$$\mathrm{a\ atom}\text{-}\sqsubseteq_i \mathrm{p}$$

if and only if a is atomic and $a \sqsubseteq_i p$.

A complete atomic semi-lattice on propositions

(31) a. Primitive set of propositions Π, as in Thomason (1980), Bealer (1982), Chierchia (1984), Aczel (1989) and Zalta (1983), for example.
b. A transitive, reflexive, antisymmetric ordering $\sqsubseteq_i$ on Π
c. For any subset π of Π, we require the "sum" of the elements of π to be another element of Π, written

$$\bigwedge_{p \in \pi} p$$

We require this proposition to be the supremum (or join or greatest lower bound or l.u.b.) of π with respect to $\sqsubseteq_i$. We also call such a proposition a *proposition sum*. Since Π is a set such that the sum of any subset is in Π, Π is a *complete join semilattice* with respect to $\sqsubseteq_i$. For notational convenience, we write

$$p_0 \sqcap p_1$$

for the supremum of p_0 and p_1.[3].
d. We define A_Π, a subset of Π, to be the set of **atoms** and to consist of those elements a such that

$$\forall p[p \sqsubseteq_i a \longrightarrow p = a]$$

We write

$$a \text{ atom-}\sqsubseteq_i p$$

if and only if a is atomic and $a \sqsubseteq_i p$.
e. We will define truth in a possible world to give "conjunction semantics" for $p_0 \sqcap p_1$. Thus, assuming possible worlds to be sets of propositions, we will require of W, the set of possible worlds:

$$\forall w \in \mathrm{W}[p_0 \in w \sqcap p_1 \in w \longleftrightarrow p_0 \sqcap p_1 \in w]$$

A Note on Atomic Propositions

(32) a. **Basic relations** will be functions from their argument types to Π, the set of propositions. The predicate *walk'*, for example, is a function from the set of entities E (following Link 1983) to Π. Thus the type theory will add to the type for truth-values t with a type for propositions p. The predicate *walk'* is of type $\langle e, p\rangle$.
b. **Atomic propositions** won't be very atomic looking. Atomic in the relevant sense just means not composed out of simpler propositions by conjunction ($\sqcap$). So all of the following rather complex propositions are atomic:
1. every'(man')(walk')
2. believe'(j')(happy'(m'))
3. ¬ [walk' (j)]

[3]Note we use $\sqcap$ rather than $\vee$, the more traditional notation for a join or supremum, because $p_0 \sqcap p_1$ corresponds to a logical conjunction of p_0 and p_1.

Types (with Boolean Types) Syntax

(33) a. p is a Boolean type
b. e is a type.
c. If b is a type and a is a type, then $\langle$ a, b $\rangle$ is a type.
d If b is a Boolean type, then $\langle$ a, b $\rangle$ is a Boolean type.

Types (with Boolean Types) Semantics

(34) a. $\mathrm{D_p} = \Pi$
b. $\mathrm{D_e} = \mathrm{E}$
c. $\mathrm{D}_{\langle \mathrm{a,b} \rangle} = {\mathrm{D}_b}^{\mathrm{D_a}}$

A hierarchy of complete atomic semi-lattices on Boolean types

For all Boolean types, $\langle$ a, b $\rangle$:

(35) a. A transitive, reflexive, antisymmetric ordering $\sqsubseteq_i$ on $\mathrm{D}_{\langle \mathrm{a,b} \rangle}$.
b. For any subset S of $\mathrm{D}_{\langle \mathrm{a,b} \rangle}$, we require the "sum" of the elements of S to be another element of $\mathrm{D}_{\langle \mathrm{a,b} \rangle}$, written

$$\bigwedge_{\mathrm{P} \in \mathrm{S}} \mathrm{P}$$

We define this individual to be the supremum (or join or greatest lower bound or l.u.b.) of S with respect to $\sqsubseteq_i$. Such a sum is called a *Boolean sum*. We write

$$\mathrm{P} \sqcap \mathrm{Q}$$

for the supremum of P and Q.
c. We define $\mathrm{A}_{\langle \mathrm{a,b} \rangle}$, a subset of $\mathrm{D}_{\langle \mathrm{a,b} \rangle}$, to be the set of **atoms** of $\mathrm{D}_{\langle \mathrm{a,b} \rangle}$ and to consist of those elements Q such that

$$\forall \mathrm{P}[\mathrm{P} \sqsubseteq_i \mathrm{Q} \longrightarrow \mathrm{P} = \mathrm{Q}]$$

We write

$$\mathrm{P}\ \text{atom-}\sqsubseteq_i\ \mathrm{Q}$$

if and only if P is atomic and $\mathrm{P} \sqsubseteq_i \mathrm{Q}$.
d. **Reduction**: For each boolean type $\langle$ a, b $\rangle$, we require of $\sqsubseteq_i$ that:

$$\forall \alpha \in \mathrm{D_a}, \beta_1\, \beta_2, \ldots \beta_n \in \mathrm{D}_{\langle \mathrm{a,b} \rangle}[\beta_1 \sqcap \beta_2, \ldots \sqcap \beta_n(\alpha) = \beta_1(\alpha) \sqcap \beta_2(\alpha), \ldots \beta_n(\alpha)]$$

8 Acknowledgements

Kehler was supported in part by National Science Foundation Grant IIS-9619126 to SRI International.

References

Aczel, P. 1989. Algebraic Semantics for Intensional Logics, I. In Chierchia, G., Partee, B., Turner, R. (eds) *Properties, Types, and Meaning* I. 17-45.

Bealer, G. 1982. *Quality and Concept.* Clarendon Press. Oxford.

Chierchia, G. 1984. *Topics in the Syntax and Semantics of Infinitives and Gerunds.* Ph.D. Dissertation. University of Massachusetts. Amherst.

Dalrymple, M. and Kehler, A. 1995. On the Constraints Imposed by *Respectively*, *Linguistic Inquiry (Squibs and Discussion)*, 26:3, pp. 531-536.

Dowty, D. 1986. A Note on Collective Predicates, Distributive Predicates, and All. In Marshall (ed) ESCOL 86. Ohio State University. Columbus.

Gazdar, G. 1980. A Cross-categorial Semantics for Conjunction. *Linguistics and Philosophy* 3. 407-409.

Kac, Michael B. 1987. Surface transitivity, *respectively* coordination and context-freeness. *Natural Language and Linguistic Theory*, 5(3):441–452.

Kay, Paul. 1989. Contextual Operators: *respective, respectively,* and *vice versa.* In Proceedings of the Fifteenth Annual Meeting of the Berkeley Linguistics Society (BLS-15), February, 1989.

Krifka, M. 1989. Boolean and non-Boolean And. Ms. University of Austin.

Lakoff, G. 1970. Linguistics and Natural Logic. *Synthese* 22. Reprinted in Davidson, D. and Harman, G. (eds) *Semantics for Natural Language* 645-665. Kluwer. Dordrecht.

Landman, F. 1989a. Groups I. *Linguistics and Philosophy* 12. 559-605.

Landman, F. 1989b. Groups II. *Linguistics and Philosophy* 12. 723-744.

Landman, F. 1996. Plurality. In Lappin, S. (ed). *Handbook of Contemporary Semantic Theory.* 425-458. Blackwell. Cambridge.

Langedoen 1978. The Logic of Reciprocity. *Linguistic Inquiry* 9. 177-197.

Link, G. 1983. The Logical Analysis of Plurals and Mass Terms: A Lattice-Theoretical Approach. In Bauerle, R., Schwarze, C., von Stechow, A. (eds). *Meaning, Use, and Interpretation of Language.* de Gruyter. Berlin.

Link, G. 1984. Hydras. On the Logic of Relative Claus Constructions with multiple heads. In Landman, F. and Veltman, F. *Varieties of Formal Semantics.* Foris. Dordrecht.

Link, G. 1991. Plural. In Wunderlich, D. and von Stechow, A. (eds) *Semantics: An International Handbook of Contemporary Research.* de Gruyter. Berlin.

McCawley, J. 1998.*The Syntactic Phenomena of English.* Second Edition. University of Chicago Press. Chicago.

Moltmann, Friederike. 1992. *Coordination and Comparatives.* Ph.D. thesis, MIT.

Pullum, Geoffrey K. and Gerald Gazdar. 1982. Natural languages and context free languages. *Linguistics and Philosophy*, 4:471–504.

Roberts, C. 1987. Modal Subordination, Anaphora, and Distributivity. Ph.D. Dissertation. University of Massachusetts. Amherst.

Rooth, M. and Partee, B. 1982. Conjunction, Type-Ambiguity and Wide-Scope *Or.* In Flickinger, D., Macken, M., and Wiegand, N. (eds) *Proceedings of WCCFL 1.* Stanford Linguistics Association. Stanford University.

Partee, B. and Rooth, M. 1983. Generalized Conjunction and Type-Ambiguity. In Bauerle, R., Schwarze, C., von Stechow, A. (eds). *Meaning, Use, and Interpretation of Language.* de Gruyter. Berlin.

Scha, R. 1981. Distributive, Collective and Cumulative Quantification. In Groenendijk, J., Janssen, T., and Stokhof, M. (eds) *Truth, Interpretation and Information.*

Schwarzchild, R. 1991. *On the Meaning of Definite Plural Noun Phrases.* Dissertation. University of Massachusetts. Amherst.

Schwarzchild, R. 1996. *Pluralities.* Kluwer. Dordrecht.

Thomason, R. 1980. A Model Theory for Propositional Attitudes. *Linguistics and Philosophy* 4. 47-70.

Zalta, E. 1983. *Abstract Objects: An Introduction to Axiomatic Metaphysics.* D Reidel. Dordrecht.